Departmental Seminar

Department of Pure Mathematics

2012 S1

Singular traces in symmetrically normed operator ideals

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Abstract: A trace of matrix is known from the basic course of linear algebra — it is a linear functional on the algebra $M_n(\mathbb{C})$ (the algebra of all $n \times n$ matrices) which is invariant with respect to conjugations. In particular, it is unitarily invariant.

In 1909, Fredholm introduced a notion of a positive functional Tr extending the classical matrix trace in the setting of compact operators on an infinite-dimensional Hilbert space H. In 1932, J. von Neumann showed that Tr is a unitarily-invariant functional on the so-called trace class $\mathcal{S}_1$ (also called the ideal of all nuclear operators in the algebra $B(H)$ of all bounded linear operators on H).

It was unknown for a long time whether there are any other traces. The first example of the trace Tr_ω was proposed by Dixmier in 1966. The trace Tr_ω vanished on the ideal $\mathcal{S}_1$. Dixmier also identified a symmetrically-normed ideal of compact operators on which the trace Tr_ω is well-defined (and continuous). Let us recall that a two-sided ideal $\mathcal{I}$ of the algebra $B(H)$ is called symmetrically normed if

1. $\mathcal{I}$ is equipped with a Banach norm $\|\cdot\|_{\mathcal{I}}$.
2. If $A \in \mathcal{I}$ and $B \in B(H)$, then $\|AB\|_{\mathcal{I}} \leq \|A\|_{\mathcal{I}}\|B\|$ and $\|BA\|_{\mathcal{I}} \leq \|A\|_{\mathcal{I}}\|B\|$.

All nontrivial ideals of $B(H)$ consist of compact operators on H.
The following question arises naturally.

Question *Which symmetrically normed ideals of the algebra $B(H)$ carry a non-trivial trace (that is, a positive unitarily-invariant functional)?*

Our main result (to appear in Crelle's journal) is as follows.

Theorem *For any symmetrically normed operator ideal $\mathcal{I}$, the following conditions are equivalent:*

1. *There exists an operator $A \in \mathcal{I}$ such that* $\lim_{n\to\infty} \frac{1}{n}\|A^{\oplus n}\|_{\mathcal{I}} > 0$.
2. *There exists a trace on $\mathcal{I}$.*

When: 12:00 Tuesday 17 April 2012
Where: Room RC-4082, Red Centre, UNSW

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